

Azimuthal asymmetries and the emergence of “collectivity” from multi-particle correlations in high-energy pA collisions

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We show how angular asymmetries $\sim \cos 2\phi$ can arise in dipole scattering at high energies. We illustrate the effects due to anisotropic fluctuations of the saturation momentum of the target with a finite correlation length in the transverse impact parameter plane, i.e. from a domain-like structure. We compute the two-particle azimuthal cumulant in this model including both one-particle factorizable as well as genuine two-particle non-factorizable contributions to the two-particle cross section. We also compute the full BBGKY hierarchy for the four-particle azimuthal cumulant and find that only the fully factorizable contribution to $c_2\{4\}$ is negative while all contributions from genuine two, three and four-particle correlations are positive. Our results may provide some qualitative insight into the origin of azimuthal asymmetries in p+Pb collisions at the LHC which reveal a change of sign of $c_2\{4\}$ in high-multiplicity events.

I. INTRODUCTION

Large azimuthal asymmetries have been observed in p+Pb collisions at the LHC [1–4] and in d+Au collisions at RHIC [5]. These asymmetries are usually measured via multi-particle angular correlations (see below) and were found to extend over a long range in rapidity. Causality then requires that the correlations originate from the earliest times of the collision [6]. Furthermore, the data shows that the asymmetries persist up to rather high transverse momenta, well beyond $p_\perp \sim 1$ GeV. Recent data by the ATLAS collaboration, for example, shows that large “elliptic” (v_2) asymmetries in p+Pb collisions at $\sqrt{s} = 5$ TeV persist up to $p_\perp = 10$ GeV [7]. Therefore, it is important to develop an understanding of their origin in terms of semi-hard (short distance) QCD dynamics [8–15].

The ALICE collaboration has measured the two- and four-particle v_2 cumulants in p+Pb collisions at 5 TeV as a function of multiplicity, see Figs. 1 and 4 in Ref. [2]. These cumulants are defined as [16]

$$c_2\{2\} = \langle \exp 2i(\phi_1 - \phi_2) \rangle, \quad (1)$$

$$c_2\{4\} = \langle \exp 2i(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle - 2 \langle \exp 2i(\phi_1 - \phi_3) \rangle \langle \exp 2i(\phi_2 - \phi_4) \rangle. \quad (2)$$

Here, $\langle \cdot \rangle$ denotes an average over the corresponding azimuthal angles weighted by the two- or four-particle distribution, respectively. The two-particle cumulant with a rapidity gap suppresses contributions from resonance decays and jet fragmentation; it depends weakly on multiplicity and is positive over the entire range of multiplicity. On the other hand the four-particle cumulant, $c_2\{4\}$, decreases monotonically and changes sign to become negative in high multiplicity events, an effect also seen by the CMS collaboration (see second paper in [4]). As shown below, this requires an anisotropy of the single-particle angular distribution. In the soft, long wavelength regime, $c_2\{4\}$ is negative when hydrodynamic flow dominates over “non-flow” correlations [17]. In this paper we perform a first computation of all connected and disconnected contributions to the cumulants in the short distance regime using a model that allows for anisotropic “domains” of the color-electric fields \vec{E} of the target [10, 18].

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II. CALCULATION

Our discussion is based on the dipole model of high-energy interactions [19]. We consider scattering of a dipole of size $r \sim 1/p_\perp$ from the target described by a particular configuration of the (color) electric field $E^i \sim F^{+i}$. For a small dipole $\vec{r} \equiv \vec{x} - \vec{y}$ the leading C-even interaction with the target is given by

$$S - 1 = \frac{1}{2N_c} \text{tr} (ig \vec{r} \cdot \vec{E})^2, \quad (3)$$

with a C-odd correction at order $(igr)^3$ which is not considered here because it does not contribute to $\sim \cos 2\phi$ asymmetries [18]. Equation (3) arises from an expansion of the S-matrix, $\text{tr} V(\vec{x})V^\dagger(\vec{y})/N_c$, in powers of \vec{r} , where

$$V(\vec{x}) = \mathcal{P} \exp \left(ig \int dx^- A^+(x^-, \vec{x}) \right) \quad (4)$$

is the path-ordered Wilson line describing the propagation of a charge in the field of the (right-moving) target. We focus on the S-matrix for a fundamental charge though the calculation could be repeated for a charge in the adjoint representation yielding the same results for $c_2\{2\}$ and $c_2\{4\}$.

To obtain the cross section the scattering matrix is averaged over the configurations of the \vec{E} field of the target. Averaging over *all* such configurations leads to

$$\langle S \rangle - 1 = \frac{(ig)^2}{2N_c} r^i r^j \left\langle \text{tr} E^i(\vec{b}) E^j(\vec{b}) \right\rangle = -\frac{1}{4} r^2 Q_s^2(\vec{b}) \log \frac{1}{r\Lambda} \quad (5)$$

in the leading log approximation, $\log 1/r\Lambda \gg 1$. Here, $Q_s(\vec{b})$ denotes the saturation scale below which non-linear effects become significant. In what follows we shall assume a very large nucleus and drop the dependence of the average saturation momentum on \vec{b} .

Equation (5) corresponds to the single-particle cross section averaged over all configurations of $\vec{E}(\vec{b})$ in the target and is, of course, isotropic. On the other hand, for any particular configuration the S-matrix does exhibit an angular dependence, c.f. for example Fig. 7 in Ref. [20]. The idea that anisotropic fluctuations of the saturation momentum would induce $v_n \neq 0$ has been presented previously in Refs. [10, 18, 21]. Hence, to evaluate the amplitude of the angular modulation of the S-matrix we perform the average subject to the constraint

$$\frac{(ig)^2}{2N_c} r^i r^j \left\langle \text{tr} E^i(\vec{b}_1) E^j(\vec{b}_2) \right\rangle_{\hat{a}} = -\frac{1}{4} r^2 Q_s^2 \log \frac{1}{r\Lambda} \Delta(\vec{b}_1 - \vec{b}_2) (1 - \mathcal{A} + 2\mathcal{A}(\hat{r} \cdot \hat{a})^2). \quad (6)$$

That is, we divide the target ensemble into classes such that for a given class the anisotropic part of the electric field correlator in the vicinity of \vec{b} (within a given “domain”) points in a specific direction. The summation over all classes, which corresponds to an integration over the directions \hat{a} , is performed only after the m -particle angular cumulant has been evaluated. The quantity \mathcal{A} in eq. (6) is the amplitude of anisotropy of the electric field correlator.

For simplicity, as we mentioned above, in our current analysis we singled out only fluctuations of \hat{a} while possible fluctuations of Q_s and \mathcal{A} are averaged out in Eq. (6). The results could be extended to account for fluctuations of Q_s and \mathcal{A} in the future.

The domain structure of the field is described by the two-point correlation function

$$\Delta(\vec{b}_1 - \vec{b}_2) = \exp \left(-\frac{|\vec{b}_1 - \vec{b}_2|^2}{\xi^2} \right), \quad (7)$$

where ξ denotes the correlation length. We assume a Gaussian correlation function, other options do not change our results qualitatively. To simplify the notation we introduce

$$\frac{1}{N_D} \equiv \frac{1}{S_\perp^2} \int d^2b_1 d^2b_2 \Delta(\vec{b}_1 - \vec{b}_2) = \frac{\pi \xi^2}{S_\perp}, \quad (8)$$

which is the area of a domain divided by the area of the collision zone, in other words, the inverse number of domains. Equation (7) essentially describes the correlations of the saturation momentum Q_s in the transverse plane.

We can now compute the angular distribution for scattering of a single dipole, for a fixed \hat{a} . Using Eqs. (6) and performing a Fourier transform to momentum space, as well as an average over the impact parameter, we arrive at

$$\left(\frac{1}{\pi} \frac{dN}{dk^2} \right)^{-1} \frac{dN}{d^2k} = 1 - 2\mathcal{A} + 4\mathcal{A}(\hat{k} \cdot \hat{a})^2. \quad (9)$$

Hence, the one-particle v_2 cumulant

$$v_2\{1\} \equiv \left\langle e^{2i(\phi_k - \phi_a)} \right\rangle_{\hat{a}} = \mathcal{A}. \quad (10)$$

To avoid confusion let us stress that here $\langle \cdot \rangle$ refers to a *different* average than the average over \vec{E} -field configurations from above; it is simply an average over the azimuthal angle ϕ_k weighted by the distribution (9).

We now proceed to two-particle distributions. The averages over \vec{E} -field configurations shall be performed assuming a Gaussian action [22] and a color diagonal four-point function although in general additional contributions could appear [10, 23]. Then the two-particle S-matrix for fixed \hat{a} is given by

$$\langle S_2 \rangle - 1 = \left(\frac{(ig)^2}{2N_c} \right)^2 \left\langle \text{tr} (\vec{r}_1 \cdot \vec{E}(\vec{b}_1))^2 \text{tr} (\vec{r}_2 \cdot \vec{E}(\vec{b}_2))^2 \right\rangle_{\hat{a}} \quad (11)$$

$$= \frac{(ig)^4}{4N_c^2} \int \frac{d\phi_{a'}}{2\pi} \left\langle \text{tr} (\vec{r}_1 \cdot \vec{E}(\vec{b}_1))^2 \right\rangle_{\hat{a}} \left\langle \text{tr} (\vec{r}_2 \cdot \vec{E}(\vec{b}_2))^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (12)$$

$$+ \frac{(ig)^4}{4N_c^2} \left\langle \text{tr} (\vec{r}_1 \cdot \vec{E}(\vec{b}_1))^2 \text{tr} (\vec{r}_2 \cdot \vec{E}(\vec{b}_2))^2 \right\rangle_{\hat{a}}^{\text{conn.}}. \quad (13)$$

The factorizable (disconnected) contribution involves the correlations of the directions of $\vec{E}(\vec{b})$ in the impact parameter plane; we employ $C(\hat{a}, \hat{a}') = 2\pi \delta(\phi_a - \phi_{a'}) \Delta(\vec{b}_1 - \vec{b}_2)$. Averaging over impact parameters gives

$$\frac{(ig)^4}{4N_c^2} \int \frac{d^2 b_1}{S_\perp} \frac{d^2 b_2}{S_\perp} \int \frac{d\phi_{a'}}{2\pi} \left\langle \text{tr} (\vec{r}_1 \cdot \vec{E}(\vec{b}_1))^2 \right\rangle_{\hat{a}} \left\langle \text{tr} (\vec{r}_2 \cdot \vec{E}(\vec{b}_2))^2 \right\rangle_{\hat{a}'} C(\hat{a}, \hat{a}') \quad (14)$$

$$= \frac{1}{N_D} \frac{1}{16} r_1^2 r_2^2 Q_s^4 \log \frac{1}{r_1 \Lambda} \log \frac{1}{r_2 \Lambda} (1 - \mathcal{A} + 2\mathcal{A}(\hat{r}_1 \cdot \hat{a})^2) (1 - \mathcal{A} + 2\mathcal{A}(\hat{r}_2 \cdot \hat{a})^2) \quad (15)$$

$$= \frac{1}{N_D} \frac{dN_1}{\pi dr_1^2} \frac{dN_2}{\pi dr_2^2} (1 - \mathcal{A} + 2\mathcal{A}(\hat{r}_1 \cdot \hat{a})^2) (1 - \mathcal{A} + 2\mathcal{A}(\hat{r}_2 \cdot \hat{a})^2). \quad (16)$$

In this expression the prefactor $1/N_D$ arises due to the fact that the orientation of the electric field is approximately constant only over distance scales of order the correlation length ξ . Multiplying the Fourier transform of this expression by $\exp(2i(\phi_1 - \phi_2))$ and averaging over the azimuthal angles leads to the disconnected (single-particle factorizable) contribution to $(v_2\{2\})^2$:

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}}^{\text{disc.}} = \frac{1}{N_D + \frac{1}{2(N_c^2 - 1)}(1 + \mathcal{A}^2)} (v_2\{1\})^2. \quad (17)$$

Note that this is independent of the global direction \hat{a} relative to which we define ϕ_1 and ϕ_2 and so the final average over \hat{a} is trivial. The additional term in the denominator originates from the connected contribution to the normalization.

The connected contribution from Eq. (13) is

$$\frac{(ig)^4}{4N_c^2} \left\langle \text{tr} (\vec{r}_1 \cdot \vec{E}(\vec{b}_1))^2 \text{tr} (\vec{r}_2 \cdot \vec{E}(\vec{b}_2))^2 \right\rangle_{\hat{a}}^{\text{conn.}} = \quad (18)$$

$$\frac{1}{8} \frac{r_1^2 r_2^2 Q_s^4}{N_c^2 - 1} \log \frac{1}{r_1 \Lambda} \log \frac{1}{r_2 \Lambda} \Delta^2(\vec{b}_1 - \vec{b}_2) [\cos(\phi_1 - \phi_2) + 2\mathcal{A}(2\cos(\phi_1 - \phi_a)\cos(\phi_2 - \phi_a) - \cos(\phi_1 - \phi_2))]^2. \quad (19)$$

Averaging over impact parameters produces a factor

$$\frac{1}{S_\perp^2} \int d^2 b_1 d^2 b_2 \Delta^2(\vec{b}_1 - \vec{b}_2) = \frac{1}{2N_D}, \quad (20)$$

so that the connected contribution to the two-particle cumulant becomes

$$\left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle_{\hat{a}}^{\text{conn.}} \equiv \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} e^{2i(\phi_1 - \phi_2)} \left[\frac{dN_2(\hat{a})}{d^2 k_1 d^2 k_2} - \frac{dN_1(\hat{a})}{d^2 k_1} \frac{dN_1(\hat{a})}{d^2 k_2} \right] / \int \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{dN_2(\hat{a})}{d^2 k_1 d^2 k_2} \quad (21)$$

$$= \frac{1}{N_D + \frac{1}{2(N_c^2 - 1)}(1 + \mathcal{A}^2)} \frac{1}{4(N_c^2 - 1)}. \quad (22)$$

As before, here the average $\langle \cdot \rangle$ on the l.h.s. is an average over ϕ_1 and ϕ_2 but does not involve averaging over \vec{E} -field configurations since the one- and two-particle distributions have already been averaged over all such configurations corresponding to a given \hat{a} . However, the r.h.s. is independent of \hat{a} so that the final average over its direction is trivial. Also, for $\mathcal{A} = \mathcal{O}(1/N_c)$ the first factor on the r.h.s. of Eqs. (17,22) can be approximated by $1/N_D$ so that in all, $v_2\{2\}$ is then given by

$$(v_2\{2\})^2 \equiv \left\langle e^{2i(\phi_1 - \phi_2)} \right\rangle = \frac{1}{N_D} \left(\mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right). \quad (23)$$

The first term is the square of the single-particle $v_2\{1\}$; it is scaled by $1/N_D$ since both particles have to scatter from the same domain. The second contribution corresponds to genuine non-factorizable two-particle correlations. Both contributions are positive; nonetheless Eq. (23) reveals the existence of two distinct regimes. For

$$\mathcal{A} \gg \frac{1}{N_c} \quad (24)$$

the ellipticity is mainly due to the asymmetry of the single-particle distribution induced by the \vec{E} -field domains. In the opposite limit

$$\mathcal{A} \ll \frac{1}{N_c}, \quad (25)$$

$v_2\{2\}$ is mainly due to genuine two-particle correlations.

Expression (23) applies when *both* particles have sufficiently high transverse momenta as we have approximated both of their S-matrices by their leading small- r behavior $\sim \text{tr}(\vec{r}_i \cdot \vec{E})^2$. On the other hand, experimentally one typically considers angular correlations of a hard with a softer particle. Recent numerical computations [24] of $c_2\{2\}$ which do not expand the S-matrices show that hard-soft correlations exhibit a fall-off with the transverse momentum of the hard particle. This is due to a decorrelation of the anisotropy axis in a high- p_T bin with that of the bulk.

The four particle cumulant exhibits qualitatively different behavior in the regimes of “small” vs. “large” \mathcal{A} . For general \mathcal{A} , $c_2\{4\}$ is given by

$$c_2\{4\} = \langle \exp(2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle - 2 \langle \exp(2i(\phi_1 - \phi_3)) \rangle \langle \exp(2i(\phi_2 - \phi_4)) \rangle \quad (26)$$

$$= -\frac{1}{N_D^3} (v_2\{1\})^4 + \langle \exp(2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle^{\text{conn.}} \quad (27)$$

$$+ \frac{1}{N_D} \langle \exp(2i(\phi_1 + \phi_2)) \rangle^{\text{conn.}} \langle \exp(-2i(\phi_3 + \phi_4)) \rangle^{\text{conn.}} + \frac{4}{N_D} v_2\{1\} \langle \exp(2i(\phi_1 + \phi_2 - \phi_3)) \rangle^{\text{conn.}} \quad (28)$$

$$+ \frac{1}{N_D^2} (v_2\{1\})^2 \langle \exp(-2i(\phi_3 + \phi_4)) \rangle^{\text{conn.}}, \quad (29)$$

which determines the azimuthal anisotropy from four particle correlations: $v_2\{4\} = (-c_2\{4\})^{1/4}$. Before addressing the corrections written in Eqs. (28,29) we compute the fully connected contribution and show that it is positive.

The fully connected contribution to the S-matrix is given by

$$(N_c^2 - 1) \prod_{i=1}^4 \frac{-Q_s^2}{4(N_c^2 - 1)} (\vec{r}_i \cdot \vec{r}_{i+1}) \Delta(\vec{b}_i - \vec{b}_{i+1}) \log \frac{1}{r_i \Lambda} + \text{permutations}, \quad (30)$$

where $i+1$ is defined modulo 4. Averaging over impact parameters generates a factor of $1/(4N_D^3)$. We may now perform the Fourier transform and sum the 48 contractions of the amplitudes / conjugate amplitudes of dipoles 1 to 4. This leads to

$$\langle \exp(2i(\phi_1 + \phi_2 - \phi_3 - \phi_4)) \rangle^{\text{conn.}} = \frac{1}{4N_D^3} \frac{1}{(N_c^2 - 1)^3} (1 + 8\mathcal{A}^2). \quad (31)$$

Here, corrections of order $\sim 1/(N_c^2 - 1)$ to the normalization have been neglected, see related discussion for $v_2\{2\}$ above. As promised, the fully connected contribution to $c_2\{4\}$ is positive; thus if the anisotropy \mathcal{A} is zero, the elliptic harmonic $v_2\{4\}$ would be *complex*. Furthermore, the magnitude of the fully connected contribution relative to $v_2\{1\}^4$ is $\sim 1/(\mathcal{A}^4 N_c^6)$. Hence, parametrically $c_2\{4\}$ crosses zero when $\mathcal{A} \sim 1/N_c^{3/2}$.

The terms from Eqs. (28,29), to leading order in N_c , are given by

$$\frac{1}{N_D^2} (v_2\{1\})^2 \langle \exp -2i(\phi_3 + \phi_4) \rangle^{\text{conn.}} = \frac{1}{N_D^3} \frac{\mathcal{A}^4}{N_c^2 - 1}, \quad (32)$$

$$\frac{1}{N_D} \langle \exp 2i(\phi_1 + \phi_2) \rangle^{\text{conn.}} \langle \exp -2i(\phi_3 + \phi_4) \rangle^{\text{conn.}} = \frac{1}{N_D^3} \frac{\mathcal{A}^4}{(N_c^2 - 1)^2}, \quad (33)$$

$$\frac{4}{N_D} v_2\{1\} \langle \exp 2i(\phi_1 + \phi_2 - \phi_3) \rangle^{\text{conn.}} = \frac{8}{3N_D^3} \frac{\mathcal{A}^4}{(N_c^2 - 1)^2}. \quad (34)$$

They provide manifestly positive contributions to $c_2\{4\}$. When \mathcal{A} is of order of $N_c^{-3/2}$, which is the regime where $c_2\{4\}$ changes sign, we can write our final result in the form

$$c_2\{4\} = -\frac{1}{N_D^3} \left(\mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right), \quad (35)$$

Here the additional terms listed in Eqs. (28,29) are suppressed by additional powers of N_c^{-2} .

III. DISCUSSION

An anisotropic single-particle distribution, $v_2\{1\} \neq 0$, requires an angular dependence of the dipole S-matrix $\sim \text{tr}(\vec{r} \cdot \vec{E})^2$ for individual configurations of \vec{E} . We describe this by the term $\sim \mathcal{A}(\hat{r} \cdot \hat{a})^2$ in Eq. (6).

Our main results are as follows. The two-particle elliptic asymmetry $c_2\{2\} \equiv (v_2\{2\})^2$ is given by

$$c_2\{2\} = \frac{1}{N_D} \left(\mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right) = \frac{1}{N_D} \left((v_2\{1\})^2 + \frac{1}{4(N_c^2 - 1)} \right). \quad (36)$$

The first term corresponds to the square of the asymmetry of the one-particle distribution while the second term is due to non-factorizable, genuine two-particle correlations. The transition between the two regimes occurs at $\mathcal{A} \sim 1/N_c$. In practice, using $N_c = 3$ and the estimate $\mathcal{A} \simeq 0.2$ from Ref. [18] we conclude that the magnitudes of both terms are comparable.

The elliptic asymmetry from four-particle correlations, $c_2\{4\} \equiv -(v_2\{4\})^4$, is

$$c_2\{4\} = -\frac{1}{N_D^3} \left[(v_2\{1\})^4 - \frac{1}{4(N_c^2 - 1)^3} \right]. \quad (37)$$

This expression applies when $v_2\{1\} = \mathcal{O}(N_c^{-3/2})$, where $c_2\{4\}$ changes sign. The first term on the r.h.s. corresponds to the fully factorized distribution and is the only negative contribution to $c_2\{4\}$. Thus, parametrically this transition to $c_2\{4\} < 0$ occurs *before* the one-particle factorizable contribution dominates $c_2\{2\}$. That is, in the vicinity of $c_2\{4\} = 0$ the two-particle cumulant $c_2\{2\}$ is dominated at leading order in $1/N_c^2$ by connected diagrams. We repeat, also, that *all* contributions in eqs. (36,37) computed within small- x QCD are long range in rapidity.

Our analysis naturally raises a question about the magnitude of the \vec{E} -field polarization amplitude \mathcal{A} and its dependence on multiplicity. Averaging over all target configurations without a multiplicity bias gives $\mathcal{A} \sim 0.1 - 0.15$ at small x [25]. In fact, $\mathcal{A}(r)$ exhibits a (weak) dependence on r at small r and this function has been found [25] to coincide with the distribution of linearly polarized gluons (for the MV model) obtained in refs. [26]. The effect of a multiplicity bias remains to be investigated. In order for the disconnected contribution to dominate in high multiplicity events, \mathcal{A} would have to grow with multiplicity.

Although our present discussion is restricted to high- p_\perp particles, i.e. small dipoles, it suggests that the measurement by the ALICE and CMS collaborations of a sign change of $c_2\{4\}$ corresponds to the fully factorizable contribution becoming dominant. The emergence of “collectivity” in pA collisions could be viewed as multi-particle correlation functions becoming dominated by fully disconnected diagrams, analogous to the BBGKY hierarchy. It will be important to understand specifically how this emerges from small- x QCD dynamics.

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